arXiv:nucl-th/9608058v1 27 Aug 1996

MULTIQUARK SYSTEMS IN A CONSTITUENT QUARK MODEL WITH CHIRAL DYNAMICS[1]

S. Pepin[2] and **Fl. Stancu**[3]

Université de Liege, Institut de Physique B.5,
Sart Tilman, B–4000 Liège 1, Belgium

and

M. Genovese[4] [5] and **J.-M.Richard**[6]

Institut des Sciences Nucléaires
Université Joseph Fourier–IN2P3-CNRS
53, avenue des Martyrs, F-38026 Grenoble Cedex, France

Abstract

We discuss the stability of multiquark systems within the recent model of Glozman *et al.* where the chromomagnetic hyperfine interaction is replaced by pseudoscalar-meson exchange. We find that such an interaction binds a heavy tetraquark system $QQ\bar{q}\bar{q}$ ($Q = c$, b and $q = u$, d) by $0.2 - 0.4$ GeV. This is at variance with results of previous models where $cc\bar{q}\bar{q}$ is unstable.

---

[2]e-mail: `u2162et@vm1.ulg.ac.be`

[3]e-mail: `u216207@vm1.ulg.ac.be`

[4]e-mail: `genovese@isnmv1.in2p3.fr`

[6]e-mail: `jmrichar@isnnx2.in2p3.fr`

The baryon spectrum has recently been revisited by using chiral models which include meson-exchange forces between quarks [1, 2, 3]. A generic Hamiltonian summarising the above references reads

$$
\begin{aligned}
H = \quad & \sum_i \frac{\vec{p}_i^2}{2m_i} - \frac{3}{16} \sum_{i<j} \tilde{\lambda}_i^c \cdot \tilde{\lambda}_j^c \, V_{\text{conf}}(r_{ij}) \\
& - \sum_{i<j} \tilde{\lambda}_i^c \cdot \tilde{\lambda}_j^c \, \vec{\sigma}_i \cdot \vec{\sigma}_j \, V_{\text{g}}(r_{ij}) + \sum_{i<j} \tilde{\lambda}_i^F \cdot \tilde{\lambda}_j^F \, \vec{\sigma}_i \cdot \vec{\sigma}_j \, V_{\text{F}}(r_{ij}),
\end{aligned} \tag{1}
$$

where m_i is the constituent mass of the quark located at $\vec{r}_i$; $r_{ij} = |\vec{r}_j - \vec{r}_i|$ denotes the interquark distance; $\vec{\sigma}_i$, $\tilde{\lambda}_i^c$, $\tilde{\lambda}_i^F$ are the spin, colour and flavour operators, respectively. Spin-orbit and tensor components may supplement the above spin-spin forces for studying orbital excitations.

The last term in H represents the meson exchange. An implicit sum over F is understood, where F = 1, 2 and 3 corresponds to π, F = 4, 5, 6 and 7 to K, F = 8 to η, and F = 0 to η'. When $V_{\text{F}} = 0$, we have a standard constituent quark model.

The confining term V_{conf} usually consists of a Coulomb plus a linear term,

$$
V_{\text{conf}} = -\frac{a}{r} + br, \tag{2}
$$

and is sometimes approximated by a harmonic potential, with possible constant terms.

The third term in H is often understood as the chromomagnetic analogue of the Breit–Fermi term of atomic physics. The radial shape of V_{g} is taken as being of very short range. For mesons, $\tilde{\lambda}_1^c \cdot \tilde{\lambda}_2^c = -16/3$, and a positive V_{g}, as in the one-gluon-exchange model, shifts each vector meson above its pseudoscalar partner, for instance $D^* > D$ in the charm sector. For baryons, where $\tilde{\lambda}_1^c \cdot \tilde{\lambda}_2^c = -8/3$ for each quark pair, such a positive V_{g} pushes the spin 3/2 ground states up, and the spin 1/2 down, for instance $\Delta > N$. In the simplest models, $V_{\text{g}} \propto \delta^{(3)}(\vec{r})$ is treated in first order. Adopting a finite-range parametrisation allows one to treat V_{g} non-perturbatively when solving the Schrödinger equation.

It has been long recognised that explicit fitting of light mesons and baryons in potential and bag models requires a large strength for the chromomagnetic term. A possible remedy is to introduce mesonic loops. For instance in the work of Myhrer *et al.* [4] or Cottingham *et al.* [5], the $\Delta - N$

1

splitting is shared between pion loops and chromomagnetism in about equal parts. The complementarity between gluon-exchange and pion-field effects arises naturally in models where the bag containing the quarks is surrounded by a pion cloud. Such a model is the "little bag" of Brown and Rho [6], where the pion field is strictly restricted to stay outside the bag. In the "cloudy bag" of Thomas and collaborators [7], the pion field is allowed to penetrate the bag.

Non-relativistic versions of the pion-exchange effect are the models introduced by Weber *et al.* [8], and by Glozman and Riska [9]. In these models, pions and other mesons are directly exchanged between the quarks, and thus travel through the very interior of the hadron.

The explicit pion-exchange contribution to the last term in Eq. (1) reads

$$\sum_{i<j} \vec{\sigma}_i \cdot \vec{\sigma}_j \, \vec{\tau}_i \cdot \vec{\tau}_j \frac{g^2}{4\pi} \frac{1}{4m^2} \left[\mu^2 \frac{\exp(-\mu r_{ij})}{r_{ij}} - 4\pi \delta^{(3)}(r_{ij}) \right], \tag{3}$$

where μ is the pion mass. A coupling constant $g^2/4\pi = 0.67$ at the quark level corresponds to the usual strength $g_{\pi NN}/4\pi \simeq 14$ for the Yukawa tail of the nucleon–nucleon (NN) potential. When constructing NN forces from meson exchanges, one disregards the short-range term in Eq. (3), for it is hidden by the hard core, and anyhow the potential in that region is parametrised empirically. Similarly, when Törnqvist [10], Manohar and Wise [11], or Ericson and Karl [12] consider pion exchange in multiquark states, they have in mind the Yukawa term $\exp(-\mu r)/r$ acting between two well separated quark clusters. For similar reasons Weber *et al.* [8] ignore the delta-term too. Therefore it is somewhat of a surprise to see the delta-term of Eq. (3) taken seriously, and with an *ad-hoc* regularisation playing a crucial role in the quark dynamics [1, 2, 3].

In the work of Glozman, Papp and Plessas [1], the chromomagnetic term is entirely omitted ($V_{\rm g} = 0$) and a weak linear confinement is supplemented by π, η and η' exchanges. The model is used to estimate the spectrum of N and Δ baryons. The explicit form of H integrated in the spin–flavour space is

$$H = H_0 + \frac{g^2}{48\pi m^2} \begin{cases} 15V_\pi - V_\eta - 2 \left(g_0/g\right)^2 V_{\eta'} & \text{for} \quad N \\ 3V_\pi + V_\eta + 2 \left(g_0/g\right)^2 V_{\eta'} & \text{for} \quad \Delta \end{cases} \tag{4}$$

2

with

$$H_0 = \sum_i m_i + \sum_i \frac{\vec{\mathrm{p}}_i^2}{2m_i} + \frac{C}{2} \sum_{i<j} r_{ij} \tag{5}$$

$$V_\mu = \Theta(r - r_0)\mu^2 \frac{\exp(-\mu r)}{r} - \frac{4\epsilon^3}{\sqrt{\pi}} \exp(-\epsilon^2(r - r_0)^2); \ (\mu = \pi, \ \eta, \ \eta'). \tag{6}$$

The parameters are $m = 0.337$ GeV, $C/2 = 0.01839$ GeV^2, $g^2/4\pi = 0.67$, $(g_0/g)^2 = 1.8$, $\epsilon = 0.573$ GeV, $r_0 = 2.18$ GeV^{-1}, and $\mu = 0.139$, 0.547, 0.958 GeV for π, η and η', respectively.

When the meson-exchange terms are switched off, the N and Δ ground states are degenerate at 1.63 GeV. When the coupling is introduced, the wave function is modified. We have performed crude variational estimates with Gaussian wave functions, and reproduced the results of the more elaborate Faddeev calculation of Ref. [1]. For the nucleon, we found that the spin-independent part H_0 of the Hamiltonian gives a contribution of 2.1 GeV, and receives a large -1.2 GeV correction from meson exchange. For the Δ ground state, the contribution of H_0 and meson exchange parts are 1.9 GeV and -0.6 GeV, respectively. Thus one ends up with a reasonable value for the $\Delta - N$ splitting, close to 0.3 GeV.

But dramatic effects occur when the model is applied to mesons or to multiquark systems containing heavy quarks. When the meson mass μ reaches values of 2 or 3 GeV as for the η_c or the D, the two terms in Eq. (3) basically cancels each other. Moreover we expect little $c\bar{c} \leftrightarrow q\bar{q}$ mixing in the η_c, so very weak coupling of η_c to a light quark q. Hence the most natural extension of the model (1) to a combination of light and charmed quarks restrict meson exchange to the former ones. Then:

1. The D and D^* mesons are degenerate. An average mass $M(D) \simeq 2$ GeV can be obtained from H_0 if the charmed quark is given a mass value of $m_c \simeq 1.35$ GeV.

2. A reasonable splitting is obtained between the isoscalar Λ_c and the (degenerate) Σ_c and Σ_c^* baryons of quark content (cqq). The masses are $\Lambda_c = 2.32$ GeV, and $\Sigma_c = \Sigma_c^* = 2.48$ GeV.

3. Another consequence is that the $(\bar{c}\bar{c}qq)$ multiquark is easily bound provided the light diquark is in a spin–isospin $S = 0$, $I = 0$ state. A crude trial wave-function (a Gaussian for each internal Jacobi coordinate) is sufficient

to give a binding energy as large as

$$2(\bar{c}q) - (\bar{c}\bar{c}qq) = 0.18 \text{ GeV}. \tag{7}$$

For $(\bar{b}\bar{b}qq)$ we obtain in the same way a binding energy

$$2(\bar{c}q) - (\bar{c}\bar{c}qq) = 0.22 \text{ GeV}, \tag{8}$$

which becomes about twice larger for more realistic potentials including a Coulomb term in the central part, as per Eq. (2).

A spin–isospin $S = 0$, $I = 0$ state implies a colour $\bar{3}$ for the qq diquark, and thus an $S = 1$, $I = 0$ and colour 3 state for $(\bar{Q}\bar{Q})$, which thus takes advantage of the confining force. It was indeed shown that in a flavour-independent potential (no hyperfine interaction) $(\bar{Q}\bar{Q}qq)$ becomes stable if the mass ratio $m(Q)/m(q)$ is large enough [13]; then, for realistic potentials, stability occurs more likely for $(\bar{b}\bar{b}qq)$ than for $(\bar{c}\bar{c}qq)$ [14, 15]. Note that the entire $\bar{Q}\bar{Q}qq$ system discussed above has $S = 1$ and $I = 0$. When the masses of $\bar{Q}$ and q are comparable, the dynamics mixes the triplet and sextet states of diquarks [16].

It was underlined by Törnqvist [10], and Manohar and Wise [11] that one-pion exchange might favour binding of heavy-flavour configurations. These authors, however, proposed quantum numbers ($S = 0$, $I = 1$) or ($S = 1$, $I = 0$) for the light diquark, so that a negative $\vec{\sigma}_1 \cdot \vec{\sigma}_2 \, \vec{\tau}_1 \cdot \vec{\tau}_2$ makes the Yukawa tail $\exp(-\mu r)/r$ attractive. This implies a colour 6 for a qq diquark with relative angular momentum $\ell = 0$, in order to fulfil the Pauli principle. Thus $\bar{Q}\bar{Q}$ is in a colour $\bar{6}$ state, at variance with the considerations above.

4. Presently we are investigating whether other multiquark systems are predicted to be stable in our simple extension of the model of Glozman *et al*. In particular we are studying the ($ccqqqq$) system. More details will be given in a forthcoming publication [17].

Acknowledgements

References

[1] L.Ya. Glozman, Z. Papp and W. Plessas, *Phys. Lett.* B **381**, 311 (1996).

[2] Z. Dziembowski, M. Fabre de la Ripelle and G.A. Miller, *Phys. Rev. C* **53**, 2038 (1996).

[3] A. Valcarce, P. González, F. Fernández and V. Vento, *Phys. Lett.* B **367**, 35 (1996).

[4] F. Myhrer, G.E. Brown and Z. Xu, *Nucl. Phys.* A **362**, 317 (1981).

[5] W.N. Cottingham, K. Tsu and J.-M. Richard, *Nucl. Phys.* B **179**, 541 (1981).

[6] G.E. Brown and M. Rho, *Phys. Lett.* **82B**, 177 (1979); **84B**, 383 (1979); *Comments Nucl. Part. Phys.* **18**, 1 (1988).

[7] A.W. Thomas, in *Adv. Nucl. Phys.*, eds. J. Negele and E. Vogt, vol. **13** (1984).

[8] M. Weyrauch and H.J. Weber, *Phys. Lett.* B **171**, 13 (1986); H.J. Weber and H.T. Williams, *Phys. Lett.* B **205**, 118 (1988); see, also F. Fernández and E. Oset, *Nucl. Phys.* A **455**, 720 (1986); I.T. Obukhovski and A.M. Kusainov, *Phys. Lett.* B **328**, 142 (1990); A. Buchmann, E. Hernández and K. Yazaki, *Nucl. Phys.* A **569**, 661 (1994).

[9] L.Ya. Glozman and D.O. Riska, *Phys. Rep.* **268**, 263 (1996).

[10] N. Törnqvist, *Phys. Rev. Lett.* **67**, 556 (1991); *Z. Phys.* C **61**, 525 (1994).

[11] A.V. Manohar and M.B. Wise, *Nucl. Phys.* B **399**, 17 (1993).

[12] T.E.O. Ericson and G. Karl, *Phys. Lett.* B **309**, 426 (1993).

[13] J.-P. Ader, J.-M. Richard and P. Taxil, *Phys. Rev. D* **25**, 2370 (1982).

[14] S. Zouzou, B. Silvestre-Brac, C. Gignoux and J.-M. Richard, *Z. Phys.* C **30**, 457 (1986); L. Heller and J.A. Tjon, *Phys. Rev. D* **32**, 755 (1985); **35**, 969 (1987); J. Carlson, L. Heller and J.A. Tjon, *Phys. Rev. D* **37**, 744 (1987); H.J. Lipkin, *Phys. Lett.* B **172**, 242 (1987).

[15] B. Silvestre-Brac and C. Semay, *Z. Phys.* C **59**, 457 (1993); **61**, 271 (1994).

[16] D.M. Brink and Fl. Stancu, *Phys. Rev.* D **49**, 4665 (1994).

[17] S. Pepin, Fl. Stancu, M. Genovese and J.-M. Richard, in preparation.